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ABSTRACT

Some diverse fields in which the effects of mental representations have been noticed are first discussed, largely through illustrations of differing problem-solving experiences. Then the LAKOS Project (Logical Analysis of Cognitive Organizational Structures) at the University of Osnabruck is described. A central concern is the specification of a model of the representation and organization of knowledge in memory. A primary goal is to describe the cognitive structures of individuals so precisely that a machine can be made to simulate aspects of their behavior. In the remainder of Part A of this paper, two cases of inconsistent student behavior in rational number learning are documented. Details from interviews are included, followed by comments. In Part B, a computer model of the mental organization of knowledge is presented which was conceptualized to understand in detail some crucial aspects of cognitive functioning and of the origins of suboptimal behavior. A tentative conclusion based on the interviews and model is then projected. (MNS)

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LOGICAL ANALYSIS OF COGNITIVE ORGANIZATIONAL STRUCTURES

Part A: The LAKOS Project

Part B: A Computer Model of Student Performance

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Part A: The LAKOS Project

1. Introduction

1.1 Motivation

In order to understand better the process of the acquisition of knowledge through schooling, it appears necessary to obtain precise descriptions and explanations of the principles how knowledge is stored, organized, and used by an individual. For those researchers regarding the acquisition of knowledge as a process of construction, the description of network structures is of central interest, in particular with respect to the way how knowledge is accessed and used and how new knowledge is integrated in existing cognitive structures.

If one restricts attention to solely cognitive behavior while leaving aside aspects of affective behavior like emotion and motive, it is near on hand to compare the human acting rationally with a problem solving system that follows the rules of logic. A problem solving system would use certain axioms and rules together with an "inference engine" to master the tasks arising from a problem situation. In many respects, a human problem solver would proceed in such a way and use his/her logical intellect to employ knowledge in order to master a situation.

But even when the logical intellect could be compared to a problem solving system that is based on predicate logic, there exist substantial differences. In general, a person has no global view of the knowledge s/he has acquired in the range of many years and cannot access every single element of this knowledge at any given moment.

Moreover, there is no guarantee that the body of knowledge possessed

by an individual is consistent. In instruction, "knowledge" is often passed on from the teacher to the student without being necessarily validated by the student. In many cases, little or no learning is involved about constraints that restrict the domain of applicability of knowledge. When taking in new information, the learner, in general, does not perform a global check in order to evaluate whether his/her knowledge base extended in this process remains consistent. Thus there is a chance that flaws exist in the knowledge of a person which are not recognized in the beginning but which are a latent factor of malperformance.

In further contrast to a computer problem solving system, human thinking is subject to certain restrictions (e.g., in the capacity of short-term memory) which can hinder more complex inferences to be carried out.

From analyses of children's behavior when using their knowledge in problem solving situations emerges the crucial role of the mental representation of their knowledge. Instead of being coherently organized, relevant knowledge structures which need to be coordinated for success in a broad range of situations appear to develop initially as isolated "packets" which are more or less restricted to the situational context in which they were acquired.

Mathematics, for example, is a field that frequently requires a coordination of diverse knowledge structures which were acquired at different occasions, in order to successfully deal with a task situation. Lack of coordination of relevant knowledge can be a reason of failure although all necessary pieces are "known".

Another salient problem in mathematics is the instability of children's performance often observed when children are to use their mathematical

knowledge in applicational situations. Frequently one finds that the standard problems in a certain field are mastered but that children stumble in contexts they have not encountered before, even though the knowledge necessary to master the situation should be available. It may then happen that alternative knowledge frameworks existing in the mind of a learner are evoked by the situation and override knowledge more adequate for the task. One effect is that the answers given to the same mathematical question posed in different contexts can differ. In this sense, the behavior of a student can be inconsistent across a variety of situations.

One attempt to shed light on the phenomenon of instable performance is the work of Thomas Seiler (1973). Seiler has conducted a series of experiments showing that juveniles already thinking formally (in the sense of Piaget) are not always able to use formal thinking operations in all problem solving tasks (not even in all tasks used by Piaget). Therefore Seiler considers it necessary to introduce a "situation and range-specific factor" in the developing cognitive structure which inhibits its generalization. Since the generalization of cognitive structures is a laborious and lasting process, Seiler proposes that the conditions and laws governing this process have to be clarified first.

In the literature, increasing indication is found supporting the fact that the phenomenon of range specifity in the development of cognitive structures is a critical issue which demands the attention of researchers and educators. The postulate that knowledge is stored in memory in discrete units has been raised by several authors and has been captured in notions like "frames" (Davis, 1980), "microworlds" (Lawler, 1981), "Subjektive Erfahrungsbereiche" ("domains of subjective experiences"; Bauersfeld, 1983) and others not mentioned here. The "microworlds" of Lawler illustrate an

idea of self-contained knowledge units which allow certain situations to be mastered but not necessarily apply to situations that slightly vary the context. Similarly, Bauersfeld (1983) and Andelfinger (1984), in comparing a number of studies in the realm of mathematics learning, postulate disparate domains of knowledge to exist in children's minds that build upon the particularity of individual learning experiences. In some respect, these findings call in question the mathematics curriculum as far as it is organized according to the logical structure of mathematics: The structures built in children's minds in the process of schooling do not seem to reflect such a logical structure.

Davis (1980) points out that everything committed to memory is stored much more permanently than was thought before ("accretion"). Access structures may change but representations are not actually deleted from memory. Where memory units concern the same topic of knowledge, this becomes a critical issue, for it will support the fact that competing knowledge may come to coexist in memory. An interpretative structure once employed by a learner to deal with certain situations, possibly erroneously, may still be present to be called upon even when more appropriate knowledge to deal with such situations has been acquired. If activated in place of the new interpretative structure, the old one may cause old mistakes to reappear.

A similar phenomenon has long been known in psycholinguistic studies on second language learning: Under certain conditions, long eradicated errors in second language tend to reemerge which appears to suggest that incorrect representations may be stored along with correct entries in long-term memory. Selinker (1972, p.215) has referred to this phenomenon as "backsliding."

A crucial issue concerns the effects of "untaught knowledge" in the sense that students can have their own interpretations ("preconceptions") prior to instruction in a certain field. For example, with respect to the order relation of rational numbers, many beginners would state that 1/3 is less than 1/4 because 3 is less than 4, an effect probably due to an overgeneralization of the order relation of whole numbers. This false belief is found to cause lasting difficulty in rational number instruction in the sense of "backsliding" (Behr, Wachsmuth, Post, & Lesh, 1984, p.333). Happs (1984, July) points out that learners have definite ideas and idiosyncratic meanings in most topics in science and asserts that such prior knowledge is utilized since the brain is apt to actively construct its own interpretations from incoming data and information already held in long-term memory.

As can be seen from the above discussion of the diverse fields in which the effects of mental representations have been noticed, attempts to characterize and understand better the nature and growth of cognitive structures represent a veritably multidisciplinary venture that faces many difficult problems. If the knowledge acquired by a student in the process of several school years is to form a coherent whole, it seems necessary, besides of identifying children's idiosyncratic conceptions, to gain insights into the principles of structuring of knowledge represented in memory and of ways to influence the development and interweavement of knowledge structures. Particular attention should be given to the reorganization of knowledge through an introduction of additional organizational structures which can bring about better access to, and better use of, the knowledge a person already possesses.

1.2 Background and objectives

Growing out of an interdisciplinary dialogue conducted for several semesters at the University of Osnabrück, a working group has been established that is aiming at further-reaching results of mutual benefit in the areas of text understanding/language production and psychologically oriented instructional research in mathematics education. In both of these disciplines, the representation of knowledge in human memory plays an essential role which has to be understood better to illuminate aspects of cognitive functioning. In the spring of 1984, a more formal collaboration was initiated to the end of developing tools that can be used in concrete projects in both disciplines, mathematics education and linguistics.

As a central concern of the LAKOS Project ("Logical Analysis of Cognitive Organizational Structures[1]"), a model of the representation and organization of knowledge in memory is being specified. The term "logical analysis" refers to the idea of capturing structures and mechanisms of knowledge organization by means of formal logic. A primary goal is to describe the cognitive structures of individuals so precisely that, based on these descriptions, a machine can be made to simulate aspects of the behavior actually observed with these subjects. This approach is commensurate with one of the original concerns of artificial intelligence. To this end, the model is specified as a computer program using the technique of logical programming (Kowalski, 1979).

The model shall help to explain aspects of human cognitive behavior when using specific knowledge in applicational situations. The emphasis of the model is on the organization of knowledge in memory. The project

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[1] Original title: "Logische Analyse Kognitiver Organisations-Strukturen"

efforts are not so much directed at identifying "misconceptions" or "alternative frameworks" per se but at modeling how they influence behavior and interact with other conceptions built under the influence of schooling. One point of interest is to model cognitive substructures that can be activated alternatively and lead to inconsistent behavior across situations. Another point of interest is to model more complex situations requiring different domains of knowledge to be coordinated on the task. The model seems to suggest that the ability and flexibility a subject possesses in this respect highly depends on the organization of her/his knowledge in memory.

In this paper we shall attempt to characterize cognitive structures as organizational structures of the memory system and explain how they can be seen responsible for irregular behaviors observed with school children. We shall view the structures of the knowledge in a person's memory to be constituted by at least the following two things:

(i) self-contained knowledge units ("packets of knowledge");

(ii) connections between these ("organizational network").

Both are the result of the individual's interaction with the outside world and internal reflective processes, and both can be subject to pedagogical interventions. In such structures is constituted the personal knowledge and beliefs of an individual, including all misconceptions and inaccuracies.

In Part A of the paper, two cases of inconsistent student behavior in mathematical situations (in the field of rational number learning) are documented, the origin of which is hypothesized to root in the unconnectedness and/or unrecognized inconsistencies of certain domains of their knowledge. In Part B, a model of the mental organization of knowledge is presented which was conceptualized to understand in detail some crucial aspects of cognitive functioning and of the origins of suboptimal behavior.

2. Descriptions of student performance in applying mathematics

2.1 Context

The notion of rational number comprises a conceptual field involving a large number of subconcepts and subaspects. Thus it constitutes a rich domain to study children's grasp and use of mathematical ideas. With respect to any such conceptual field, Vergnaud (1983) points out the importance of obtaining insights into children's use of mathematical knowledge in applicational situations, since the knowledge to be learned has to be relafed to situations for which this knowledge is "functional." In a series of studies conducted by the Rational Number Project (see acknowledgement) situations were constructed that did not expressively call for, but required a coordinated application of, several subconcepts of rational number in order to succeed (cf. Wachsmuth, Behr, & Post, 1983, April).

One of these studies is the "Gray Levels Study" which was conducted after completing 30 weeks of experimental instruction. In video-taped one-on-one clinical interviews, sixteen 5th-grade subjects were presented with a complex problem solving task. The task involved a set of 12 fractions, written as symbols a/b on little cards, which were said to represent ink mixtures with a parts black ink in b parts solution. The fractions were to be ordered by size and to be associated with stages on a scale of 11 distinct gray levels arranged by increasing "grayness" from 0% (white) to 100% (black) in stages of 10% . Presented were the fractions 0/20, 1/5, 2/7, 6/20, 2/5, 4/10, 6/15, 2/4, 4/8, 4/6, 6/9, and 12/15.

Although the visual information could be used as a guidance, it was necessary to use the numeric information and apply fraction knowledge in order to perform well on the task. Requiring the coordinated application of

a broad scale of relevant skills, such as the recognition and production of equivalent fractions, the gray levels task was expected to elicit how children bring their rational number knowledge to function in a complex applicational situation.

Three of the sixteen children were so successful that the average deviation in their card displacement was less than half a stage off the correct location. Other children were much less successful (cf. Wachsmuth et al., 1983, April). The differences are assumed to be due to different ability in activating relevant domains of fraction knowledge and coordinating it on the task.

From the data obtained in the gray levels study, selected interview material is presented here for an analysis of characteristical features of mental representation structures of children's knowledge and of cognitive mechanisms acting on such representation structures. With respect to the activation and coordination of relevant knowledge, two children seem particularly interesting: a very low performer, Terri, and a close-to-perfect-but-not-perfect performer, Bert (not their real names). The observations of both can serve to generate hypotheses about lacks in their cognitive structures impeding better performance. Observations with other subjects are used to back-up these hypotheses.

2.2 A dialogue with Bert

The point we want to emphasize first is that knowledge possessed by an individual is not necessarily available in an applicational situation demanding increased cognitive attention. In his performance on the gray levels task, Bert, in general a relatively high-achieving subject, exhibits inconsistent behavior in the following respect. In the beginning Bert

recognizes the equivalence of the fractions 4/6 and 6/9, and also of 2/5, 4/10, and 6/15. At this time, he is able to infer that equivalent fractions should be associated with the same gray value. However, in the course of working the problem, Bert associates the fractions previously regarded as equal with different but adjacent gray levels at about the correct location. That is, independently of his knowledge about fraction equivalence, Bert exhibits a good perception of fraction size. When he is asked about the fractions in question after completing the whole task, he realizes his mistake and corrects it. This is further commented on in the following dialogue excerpt.

0. BERT: (Early-on, sorts the cards and puts 2/4 and 4/8 together on table.)
1. INTERVIEWER: You put two-fourths and four-eighths together?
2. BERT: (picks them up) They're equal.
3. INTERVIEWER: I see... Would you put them on the same card (i.e., gray level)?
4. BERT: Yeah... (now puts 6/9 together with 4/6) These two are equal...

That is, before Bert starts putting cards at the gray level scale, he makes some observations about the fractions and only then puts them, one-by-one, at the scale. In so doing, he puts 4/6 at the 60% level and 6/9 at the 70% level. Similarly, he puts 2/5 at 40%, 4/10 at 45%, and 6/15

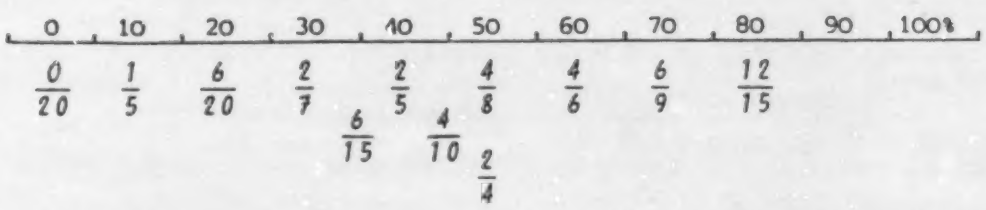

Figure 1

at 35% (see Figure 1; the percent marks were not present on the gray level scale). That is, with respect to placement on the scale, Bert rates these fractions as very close but has lost sight of their equivalence.

5. INTERVIEWER: (after the whole task has been completed) You put six-ninths right of four-sixths, why did you do that?
6. BERT: Because four-ninths-and-a-half would be half a unit...

Bert appears to be talking about 4 1/2-ninths which indicates that he has employed a self-developed strategy well-known from earlier observations. In comparing the size of fractions, Bert frequently used 1/2 as a point of reference. (For example, he would find that 4/7 is less than 3/5 by the following argumentation: "4-sevenths; three and a-half sevenths would be half a unit and 4-sevenths is half-a-seventh over; 3-fifths; two and a-half fifths would be half a unit and 3-fifths is half-a-fifth over; and one-half seventh is less over (1/2) than one-half fifth, so 4-sevenths is less than 3-fifths.")

7. INTERVIEWER: ... Before, you mentioned that they are equal .. four-sixths and six-ninths ...
8. BERT: Oh yeah, they are! (picks up 6/9 and 4/6) I think they'd be right there (puts both cards on 60%).

.....

9. INTERVIEWER: What did you think when you put 6/20 (points at 20%)?
10. BERT: Because six-twentieths is greater than one-fifth; one-fifth equals four-twentieths.

.....

11. INTERVIEWER: You put two-fifths there (40%) and four-tenths there (45%) What was your thinking?

12. BERT: Well, four-tenths would probably be ... well ... they're equal! (laughs, puts 4/10 over 2/5 on 40%) I didn't notice this.

It appears that optimal performance would involve that a subject is able to coordinate his/her knowledge about fraction equivalence with solid strategies on ordering fractions. Bert's behavior suggests that he possesses both of these relevant knowledge structures and sometimes is able to coordinate them (line 10), but the connections are still somewhat latent in his performance on the task.

Even without making use of all equivalences, Bert's placement of the cards was considerably close to correct. This phenomenon of a good sense of fraction size independent of recognition of equivalences is displayed similarly in the performance of four other "high" subjects, in that they all placed 4/6 and 6/9 at different but adjacent gray levels close to the correct position. Notably, in an interview conducted about one-half hour later under a different format (ratio symbols were used in place of fractions, e.g., 2:3 in place of 2/5, etc.), Bert displayed similar behavior in that he put 2:3, 4:6, and 6:9 at different but adjacent gray levels.

With respect to Piaget's stages of cognitive development, Bert (age 10;11;24) could be considered transitional from the concrete to the formal-operational stage. In an earlier interview assessing children's ability to compare pairs of fractions and pairs of ratios presented in a symbolical form (cf. Wachsmuth, Behr, & Post, 1983), Bert had mastered each of 18 (2 x 9) tasks of varying difficulty. Thus, the above document of inconsistent student behavior seems suited to illuminate some critical aspects about Bert's developing cognitive structure with respect to the range specifity of his rational number knowledge.

2.3 A dialogue with Terri

Terri (age 11;6;24) was a low-achieving subject who was observed to have severe difficulties impeding successful learning. Rather than building coherent knowledge structures guided by the instruction, Terri was likely to invent her own, often flawed, "theories" and procedures. Interventions in classroom instruction sometimes made her arrive at an "insight" which could turn out to have been an ephemeral one the very next day. At the time of the interview, the interviewer had known Terri from daily classroom contacts and other interviews for more than one year and was quite familiar with her idiosyncratic styles of thinking.

In working the gray levels task, Terri arrived at the following solution (Figure 2).

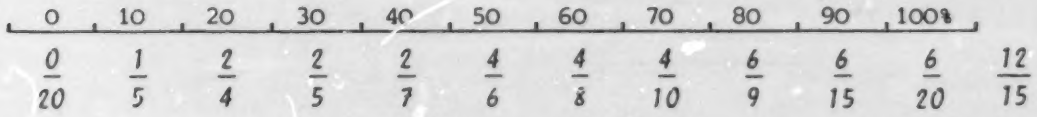

Figure 2

In the commented transcript from the interview following her solution of the gray levels task we try to pinpoint some of the inconsistencies and misconceptions in Terri's rational number knowledge. Her behavior seemed to indicate that two competing knowledge substructures served as bases for her decisions on comparisons of fractions she was presented with. From earlier observations, Terri was known to persistingly call pairs of fractions - when presented to her as written or spoken symbols - equivalent if they had the same denominator.

In contrast to this, Terri had now attached the fractions 6/15 and 12/15 to different gray levels (90% and right of 100%), apparently follow-

ing some kind of "lexical" ordering bearing on the whole number symbols in the 12 fractions. This fact raises doubts over whether Terri had understood at all the interpretation of fraction symbols by means of gray levels. However, at least in the beginning it seemed to have been clear to her as will be seen from the following dialogue. After she has placed all fraction cards at the gray level scale, Terri is asked at first why she has put 0/20 at the beginning of the scale (white, i.e. 0%). Terri explains:

0. TERRI: Because there'd be no black ink, no black ink
so it would be clear water.

After a short dialogue about 4/8 and 4/6 which Terri calls about equally dark, but 4/8 still a little bit darker than 4/6, Terri is asked about the two fractions 6/15 and 12/15.

1. INTERVIEWER: Now, Terri, what about 6/15 and 12/15?
2. TERRI: They're equal, like (laughs).
3. INTERVIEWER: OK, but you put them in different positions, though, why did you do that?

Note that by this question, Terri's attention is called back to the task situation where she, not necessarily through an interpretation in terms of gray levels but presumably through her strategy of "lexical ordering", has rated 6/15 and 12/15 as being different. This is in contradiction to her momentary opinion that these two fractions are equal. She responds:

4. TERRI: Because! That's the way I thought I should do it!
(moves and messes up chart).

Presumably, confrontation of her current opinion with her previous one results in a cognitive conflict which Terri apparently is trying to escape

from by destroying the solution she constructed. After a short dialogue (Terri should have been asked more questions about other cards) the interviewer continues (without Terri's solution being further displayed):

5. INTERVIEWER: I would still like to know: you say six-fifteenths and twelve-fifteenths are equal?

The interviewer returns to this question to find out why Terri has earlier called the fractions equal; besides, he is interested now in which of her opinions will persist through the conflict.

6. TERRI: Right.
7. INTERVIEWER: But you put them on different parts...
8. TERRI: 'Cause six comes before twelve so I thought that's the way you do it...

Terri's response confirms the impression that she had in mind, without making any connection to the gray levels, a lexical ordering strategy guided by the whole number relationsships in the fraction symbols. Now the interviewer wants to find out whether gray levels have played any part at all in her doing. (Remember that in the beginning, Terri had explained her placement of the 0/20 card by making reference to gray levels).

9. INTERVIEWER: OK, did you think in terms of darkness when you did that?
10. TERRI: Yeah, sorta like...

Terri's answer does not sound convincing. Even when gray levels have played a part in her placing of 0/20, one is tempted to assume that she had focused on a whole number ordering strategy. The next question is to find out whether Terri, in the situational context of gray levels, realizes that

12/15 represents a darker mixture than 6/15 does.

11. INTERVIEWER: Which would be darker? Six-fifteenths or twelve-fifteenths?

12. TERRI: Twelve-fifteenths.

She does rate 12/15 as darker than 6/15, but can she reach a conclusion on the ordering of the fractions 6/15 and 12/15 from this?

13. INTERVIEWER: OK, and which fraction would be bigger?

14. TERRI: Twelve-fifteenths.

Terri apparently infers that 12/15 should be the greater fraction of the two. This inference is based on an interpretation of the fraction symbols which grounds its meaning on gray levels, but it already states a "greater" (and no longer "darker") relationship between the two fractions. The inferred statement, however, continues to be in conflict with Terri's earlier opinion about the relationship between 6/15 and 12/15 which apparently resulted from her flawed "theory" of when two fractions should be equivalent (i.e., when presented to her in a purely symbolical context) Terri calls same denominator fractions equivalent).

The interviewer's next question is to find out whether Terri's opinion inferred meaningfully (12/15 greater than 6/15) outweighs her earlier opinion which was based on her "theory" of symbolical fraction equivalence. A critical section in the interview begins here. Through careful wording (namely, as it had been used in a stereotypical fashion in repeated interviews presenting fraction comparisons in a purely symbolical setting[1]), the interviewer on purpose attempts to trigger Terri's "theory" of symbolical

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[1] The original wording in these interviews was like "One-fifth and one-sixth, are they equal or is one less? - Which one is less? - Tell me how you know!"

fraction equivalence.

15. INTERVIEWER: And if I ask you, six-fifteenths, twelve-fifteenths, are they equal or is one less?

Note that if Terri's "theory" were activated by these key words, she should reply, 'They are equal'.

16. TERRI: It's less.

I.e. one is less, that is, Terri does not call them equal which is (surprisingly at the moment) not the answer anticipated from her "theory". Should the conclusion inferred on the basis of gray levels have ultimately affected Terri's belief? The interviewer's next question ('which one is less?') is posed even though Terri has already named 12/15 as the greater fraction. This question corresponds - in wording and in the sequence of events - to the stereotypical situation of the interviews on symbolical fraction comparisons and thus again addresses (as is the interviewer's hypothesis at this point) Terri's "theory" on equivalence of fractions presented to her symbolically.

17. INTERVIEWER: Which one is less?

18. TERRI: Six... um... fifteenths.

And now the interviewer wants to know which of Terri's theories her opinion, after all, is based upon.

19. INTERVIEWER: And why did you say it's less?

20. TERRI: 'Cause it... oh! (puts head in hand and sighs) No, they're equal. Because they have the same denominator.

3. Observations

We observe several instances of the critical part which cognitive structures and mechanisms play in Bert's and Terri's behavior.

1. Relevant knowledge can remain latent in a task situation, i.e. the subject "knows" but does not access particular facts which apply to the situation. This is observed with both Bert and Terri. That is, their "current opinion" is not based on the global knowledge they have acquired which is relevant to the task. Were it so, Terri would have to become aware herself of the inconsistencies existing in her knowledge, and Bert would have to maintain his opinion about the equivalences recognized. We can rather conclude that the "current opinion" of the subjects is based on a local subset of their knowledge, depending on what they are currently focussing on.

2. Another point is the possible lack of mutual accessibility of relevant knowledge units, e.g., in the context of one the subject may not always be able to access another one. While a rather clear-cut distinction in Terri's behavior indicates a disparity in her knowledge, Bert's behavior gives rise to the assumption that distinct knowledge units do exist in his mind (i.e., an "equivalence unit" and an "order unit") but that mutual access is partly developing. This is yet another instance in support of the fact that knowledge tends to develop in discrete units and that attention has to be given to the development of a proper access framework.

3. We mention the critical role of language cues (and of other cues possibly generated from a situation). As is shown in the dialogue with Terri, certain language can serve to shift the subject's focus to access knowledge contained in other memory units while loosing sight of knowledge

contained in memory units accessed previously. In a striking instance this is documented in lines 15 - 20 of the transcript of the dialogue with Terri. The current opinion of Terri (lines 16 and 18) is obtained by the chain of inferences she has gone through before (lines 11 - 14) and is supported by the meaning constructed from the situational context of gray levels. Apparently, the resulting conclusion (6/15 less than 12/15) is still present in Terri's short-term memory while the chain of inferences which made her arrive at this conclusion is no longer present in her short-term memory. But then the interviewer, again, calls for reasons while cueing her knowledge on symbolical fraction equivalence (line 19). Indeed, Terri's focus turns out to have shifted back to this realm: In order to give a reason, Terri has to make a new inference, based on her current focus. And - no way out of there - she comes up (line 20) with an according opinion (changed again!), together with an appropriate reason.

4. Cognitive restrictions can limit the use of relevant knowledge a subject possesses and can possibly intercept the change of incorrect beliefs. Regarding Terri, one is tempted to resign on the usefulness of a socratic style of dialogue and on whether incorrect beliefs of a subject can be changed through such a dialogue. Admittedly, the example discussed is an extreme one and probably requires further analysis in terms of attitudinal patterns in the interaction of interviewer and subject. It shows, however, that a single intervention does not necessarily lead to an "insight" which becomes persistent instead of being a momentary one. Presumably, the access structures calling on Terri's flawed knowledge on symbolic fraction equivalence are much stronger than the connection made on the basis of a several-step inference which the subject is not likely to achieve all by herself. A point can be made that a more global consistency

check and revision of acquired knowledge structures requires cognitive capabilities this child has not developed so far. A momentary and single "insight" is not sufficient for a long-term change in the cognitive structures manifested in Terri; it will need more than that.

4. Conclusion

In Section 1.2, we made a distinction between units in which knowledge is stored and the global organizational structure established by the connections between such units. It is one thing that a subject can have acquired incorrect knowledge (e.g., Terri's flawed theory on fraction equivalence); it is yet another thing that relevant knowledge, whether or not it is correct, does not become activated in a situation when it should be. Moreover, the fact that incorrect as well as correct knowledge on the same topic can coexist in memory calls for particular attention on how instruction can help to improve access to the right piece of knowledge at the right time. The study of mental representation structures appears a central issue in this respect.

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Part B: A Computer Model of Student Performance

1. Introduction: Computer simulations of intelligent behavior

The central idea in the theoretical study of "artificial intelligence" (AI) is "to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it" (original wording of Rockefeller proposal for the Dartmouth conference in 1956; cf. McCorduck, 1979). Following the paradigm that the human is an information processing system, AI is the study of how to organize processes to bring about "intelligent" behavior.

The so-called information processing approach in cognitive psychology is based on this paradigm: The human is regarded as a system that takes in and processes information, and human behavior is interpreted as the result of such processing of information. Information processing models constitute an approach of extending Piaget's research questions (How is knowledge structured at different stages of cognitive development?) to reach for an understanding of the process of change of cognitive structures which occurs as a result of an individual's active interaction with the outside world. The basic idea of modeling cognitive processes in the computer is that "learning to generate is learning to understand." Rigid restrictions imposed on computer simulations of cognitive processes require that not only the product, i.e. the "intelligent" behavior exhibited, but also the processes giving rise to such behavior resemble those observed with human performers.

The computer simulations of human problem solving processes carried out by Newell and Simon (1972) in the 60's/70's have demonstrated the

possible ways in which AI can contribute to an understanding of cognitive processes. An example where such approaches have been exploited in the domain of mathematical education is the work of Briars and Larkin (1984). They have compared computer simulations with empirical data from children solving elementary-level word problems and have conceptualized computer models of different complexity to explain different levels of problem solving skill.

In contrast to behavioristic approaches, the main advantage of the information processing approach lies in a detailed analysis of problem solving processes that makes specific assumptions about components of mental processes. A central role is attributed to the way in which knowledge is represented in memory. It seems that problem solving skill depends to a great extent on problem understanding as supported by the quality of mental representation and organization of knowledge. Thus the field of "knowledge representation" is viewed as crucial for obtaining insights into formal knowledge and thinking structures.

An information processing model is commonly formulated as a computer program as is the case with the model to be presented in this paper. The rationale for this lies in the complexity of human behavior; even a model restricted to only some aspects must reflect this complexity to a certain extent (cf. Briars & Larkin, 1984). It is not claimed that information processing models can capture the richness of human behavior in every aspect that can be thought of. Even when leaving aside affect and motive, the modeling of cognitive behavior poses complicated problems as is explained in more detail in the sections to follow. Under the assumption that cognitive behavior is produced by the complex interaction of identifiable laws, the computer proves to be a tool for generating predictions on

the kinds of behavior possible with an accuracy hardly reached by other means. This is the guiding idea in the approach presented here.

2. The Computer Model

In the remainder of this paper, we present an information processing model which was conceptualized to explain aspects of the cognitive behavior of human beings when using specific knowledge in applicational situations. The model is specified in the form of a computer program in the Prolog language. Prolog ("Programming in logic") is used in artificial intelligence in the domain of knowledge representation and for constructing expert systems. It embodies the concept of a "theorem prover" that works with a knowledge base consisting of axioms (facts and rules). The model is implemented in MLOG 2 (Gust & Gust, 1984) which is a highly interactive Prolog system developed for use wih microcomputers based on the Z80 processor. A first running version of the model is called LAKOS.1.

LAKOS.1 is a system which can perform natural language dialogues of a restricted, standardized form with a user. Thereby the user proceeds by asking questions or probing behavior in a similar way as in a diagnostic interview. The computer takes the role of an individual, some rudiments of which are modeled in the machine, and answers questions or executes commands from the individual's point of view. The system's responses are displayed on the terminal. They represent the actions or answer statements of the individual as predicted by the model.

It is important to note that the way how the system reacts to an input does not consist in a choice from a range of pre-programmed answers (in the sense of stimulus-response behavior). Rather, a reaction is generated ad-hoc as a knowledge-based process. That is, the actions carried out by the

system is given rise to by its body of knowledge. The potential actions of the system depend on the organizational structuring of its knowledge base. Depending on the course of dialogue, context and wording, it is possible that inputs connoting the same "meaning" are responded to by different, possibly inconsistent, reactions.

In the following, a short description of the system is given. More details are presented in Section 3 where an application of the computer model is demonstrated. As is seen in Figure 1, the system consists of a user interface, a knowledge base referred to as long-term memory, and three processes, PARSE, EVALUATE, and RESPONSE, which constitute components in the cognitive processing carried out by the system. In terms of an expert system architecture, EVALUATE could be termed the "inference engine" of the system. Further components of the system are a semantic short-term memory and a mechanism regulating the activation of knowledge coded in long-term memory referred to as focus. (In Figure 1, double arrows represent the flow of information in the system when processing an input, and simple arrows indicate where stored information is accessed.)

The system accepts standardized natural-language inputs and is communicated with via the terminal. A question or command statement posed to the system must first be understood by the system. This is accomplished by the process PARSE which attempts to decompose an input sentence to obtain a structured symbol string representing its meaning in a form the system can deal with. We use the term semantic representation to refer to such a symbol string. When attempting to construct a semantic representation of an input sentence, PARSE accesses lexical language records in long-term memory. In this process, a subset of the knowledge recorded in long-term memory is activated.

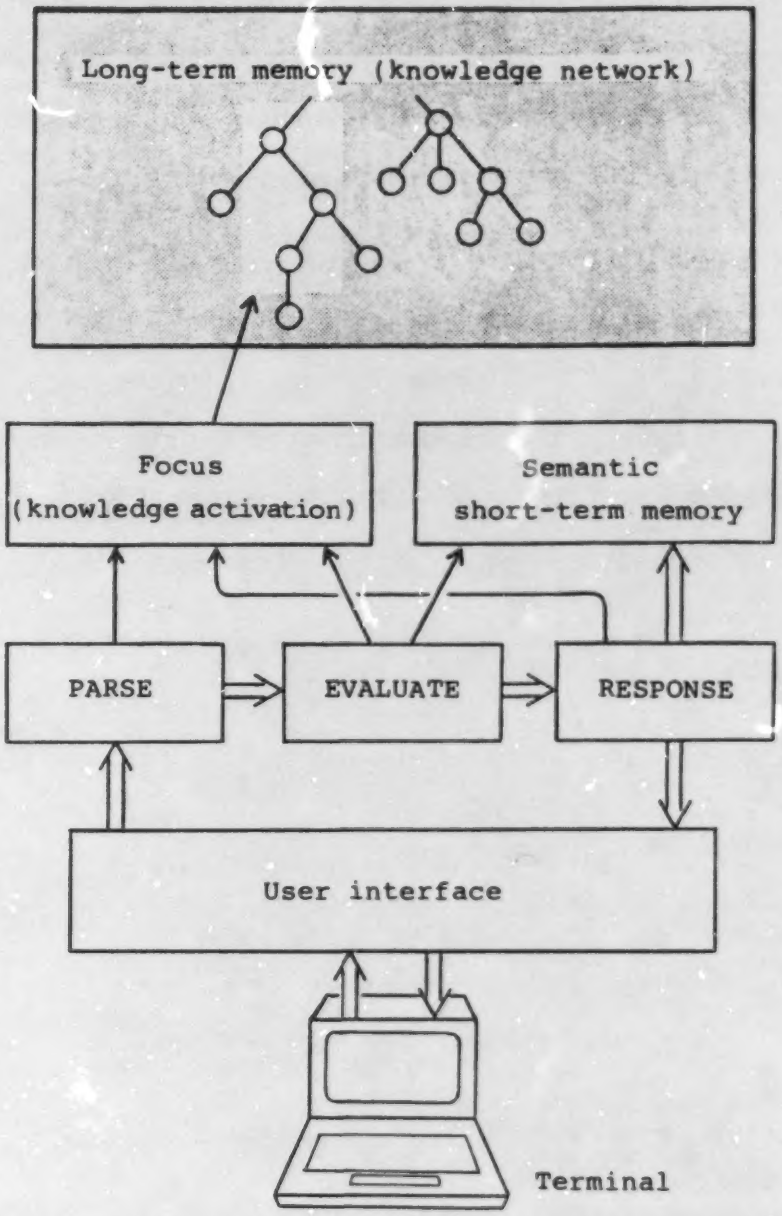

Figure 1 The system LAKOS.1

If PARSE is successful ("finished"), the process EVALUATE will search the activated part of the knowledge base for relevant information to process the input and use this information in order to produce an answer, more exactly, the semantic representation of an answer.

If the evaluation process is successful, the process RESPONSE seeks to generate a language answer and returns it to the terminal. The semantic representations of a bound number of answers produced in the course of a dialogue, together with the representations of the corresponding questions, are kept in short-term memory. That is, the results of inferences just made are available for possible use in the evaluation of further questions.

The knowledge in long-term memory is organized in the form of a network which we refer to as the knowledge network. The nodes of this network typically contain two types of records. Firstly, they contain lexical language records, i.e., symbols with associated meanings in a form usable by the system. Secondly, the nodes contain knowledge of a particular field of discourse in the form of abstract rules. These rules can be interpreted as abstract ways to think and act (similar to the notion of schema as used by Piaget). We use the term knowledge element to refer to a single record in a node of the knowledge network. A knowledge element can be employed when it is marked active and when the data or part of the data of an input string match its structure.

From the way the nodes are interweaved to form a network, the knowledge structures (i.e. the interdependencies of the schemata) of the model are constituted. As was said above, access to the knowledge in long-term memory is subject to restrictions: Only knowledge activated in the given situation can be employed. The activation of knowledge is realized through

a focus mechanism which tags the knowledge elements currently accessible. The focus can shift during a dialogue is in progress. From this, a dynamic partitioning of the knowledge network in active and in inactive knowledge is resulting.

The first version of the LAKOS model assumes a tree-like structuring of the knowledge network. On the basis of this model version, the concepts of activation, accessibility, and range specifity of knowledge can be introduced in a precise way, and simple processes of understanding can be simulated. An advanced version currently in progress will take into account general lattice structures, and a multi-focus mechanism shall allow to model more complex processes of understanding. Future versions are planned to be capable of knowledge acquisition and reorganization on their own to allow for processes of learning to be modeled. The current version constitutes a model which (at run-time) is static with respect to its knowledge and is utilized from the aspect of reproduction of behavior allowing the study of psychological segments of intellectual behavior in the simulation.

Based on empirical analyses of processes of the acquisition and use of specific knowledge (in mathematics), first applications of the model are aimed at explaining aspects of behavior exhibited by individual school children in rational number situations. The empirical material used stems from a long-term experimental teaching study conducted in 1981-83 in the USA (the "Rational Number Project"). Formatted transcripts of videotaped documents, backed up by daily observer notes (see part A), served as a basis for an instantiation of the model that simulates behavioral aspects of a particular individual. Though the points raised pertain to the realm of rational number learning, the insights gleaned from these examples are believed to be of general relevance to mathematics and science learning.

3. An application of the model to explain mathematical behavior

In the following sections is described how the computer model can be used to explain the mathematical behavior of a student in an applicational situation. Thereby, the emphasis lies on the modeling technique.

3.1 Inconsistent student behavior in applicational situations

In Part A of this paper, we have discussed the phenomenon of range specifity of knowledge. It is hypothesized that this phenomenon can be the origin of instable student performance when using mathematics in applicational situations - instable performance which does not arise from misconceptions in the first place but from the fact that the domain in which certain knowledge can be applied is constrained. The question is whether the knowledge is instable itself or whether the activation of knowledge is subject to variations. We claim that the second is the case and employ the computer model to elucidate the conditions giving rise to such variations.

The idea of using a cognitive model to explain behavior is, basically, to find rational ("logical") explanations for behavior which appears to be irrational. Using the LAKOS model and its conceptual framework, processes of knowledge utilization can be analyzed and described in a concise way. Particular aspects can be simulated for making the analysis more precise.

In Part A, we have used interview excerpts for an analysis of characteristical features of two 5th grade children's mental representation structures and of cognitive mechanisms operating on such structures. We found indication for several factors inhibiting an optimal use of knowledge in an applicational situation, such as the latency of relevant knowledge and lack of mutual access.

For example, from the interview with Terri, we drew the conclusion that two competitive domains of knowledge specific to certain situations can be identified. These domains compete with respect to certain types of problems which can be dealt with on the basis of each. Triggered by contextual cues, each can be activated independently of the other, however, a connection across the different situations is lacking. When contradictory answers given in different contexts are contrasted in the interview, the inconsistency in Terri's knowledge is revealed and a conflict is resulting.

The LAKOS model is capable of generating some aspects of such behavior based on knowledge structures hypothetically specified. We use the case of Terri to present further facts about the model and to demonstrate its use.

3.2 The TERRI program

TERRI is a program which models the hypothesized knowledge structures of the student Terri in a restricted domain of rational number knowledge (size comparisons of fractions). The program consists of the system LAKOS.1 with an instantiated knowledge network. Aspects of behavior actually observed with Terri can be simulated by conducting a dialogue in the style of a diagnostic interview with the computer. Thereby, the machine takes the role of Terri. Using the keyboard, a user of the model can pose questions or command statements to TERRI. The responses of the system appear on the screen in the form of propositions which represent statements or actions that are derived from TERRI's knowledge base. By asking WHY? the computer model can be made to give a reason for its last answer.

The answers of the model are not necessarily consistent but can vary depending on the current context, wording, and part of the history of the course of dialogue. It is possible that conflicting answers are recognized

by the system if they are appropriately contrasted in the dialogue.

The main effort in implementing the system was devoted to modeling Terri's knowledge structures. Less attention was given to the standardized form of language (a matter requiring technical efforts), as long as TERRI's answers are inambiguous.

3.3 How the model works

A simple example shall demonstrate how the model works and serve to explain more technical details. Given a question like "1 / 4 and 1 / 3 , which is less?" TERRI would respond that the second fraction is less from the fact that the numerators are equal but that 3 is less than 4. In the current version of the model this comes about as follows (by the system prompt, I'M TERRI› is indicated that the system is currently acting on the basis of Terri's hypothetical knowledge and is awaiting an input):

```
I'M TERRI› 1 / 4 AND 1 / 3 , WHICH-LESS?
   1 / 4 AND 1 / 3 , SECOND-LESS! _____ FOCUS (F 2:*1)
I'M TERRI› WHY?
   BECAUSE (EQ 1 1) (LESS 3 4)! _____ FOCUS (F 2:*1)
```

Part of the answer generated by the system indicates in which focus TERRI has answered the question. "FOCUS (F 2:*1)" means that a subnet of TERRI's fraction knowledge was activated, which in this case consists of a single node, (F 2). This node contains, or has access to, language records giving rise to the construction of a semantic representation of the question. It also contains rules whose premises match the representation of the question, in which the actual data could be embedded, and by which an answer to the question was given rise to. In this sense, (F 2) constitutes

an "adequate knowledge representation structure" as postulated by Davis, Young, and McLoughlin (1982, p.119).

An example of a language record is given next.

(SEM *CURRENT-FOCUS TALK WHICH-LESS (OR < >))

This means that the PARSE process, in constructing a representation of a language statement replaces the expression WHICH-LESS by the string, (OR < >). In this case the node index is unspecified (note that variables are marked by *), that is, this language record is "visible" in any current focus. The semantic representation of the first question in the above dialogue reads as follows:

(? ((OR < >) (1 4) (1 3)))

That is, the system will evaluate WHICH-LESS by testing whether the left term is less than the right term or whether the right term is less than the left term.

An example of a rule is given next.

(SEM (F 2:*1) RULE TRUE (< (*X *Y)(*U *V)) ⇐ (EQ *U *X)(LESS *Y *V))

This means that, focussing on node (F 2) or any subordinate node (i.e., a node whose index begins with "F 2"), TERRI will find that one of two fractions with same numerators to be less which has the smaller denominator.

A working cycle of the system consists of three major steps (cp. Section 2 and Figure 1): (1) the construction of a semantic representation of an input statement (PARSE), (2) th search and application of knowledge to construct a response internally (EVALUATE), and (3) the synthesis of an

according language response (RESPONSE). Only the active part of TERRI's knowledge base can be used in this process where it depends on the current focus which knowledge is accessible to understand and answer a question.

If the process fails at any one of the three steps, an according message is put out. If PARSE fails (i.e., the system cannot interpret the input), the response is: DON'T UNDERSTAND! If EVALUATE fails (i.e., the input was understood but no knowledge to answer the question was found), the system responds by DON'T KNOW! If finally RESPONSE cannot produce an answer statement for a finished evaluation (i.e., the system "knows" an answer but is lacking words to express it), the system puts out: CAN'T SAY! In this case the semantic representation of the answer can be inspected by listing the contents of the short-term memory.

3.4 Modeling structures of knowledge and access

As the basis for the model it is assumed (see Section 1.2 in Part A) that the individual structures of human memory are constituted by (i) self-contained knowledge units (packets of knowledge) and (ii) connections between these (organizational network). In the terminology introduced in Section 2, we say "knowledge network" to refer to this kind of a knowledge structure. A knowledge unit is comprised by a single node or by a subnet consisting of several nodes.

The central idea of this modeling approach is that the potential actions an individual is able to perform are determined by his/her knowledge network. The power of the computer model thus lies in the fact that its actions in the course of a simulated dialogue is given rise to by the organizational structuring of its knowledge base.

The modeling of behavioral aspects of Terri (the individual) in the TERRI program was accomplished as follows. Based on protocol analyses of clinical interviews, the hypothetic knowledge structure of Terri concerning size comparisons of fractions was modeled in a network. The knowledge of Terri was grasped in single elements each constituting a modular piece of Terri's knowledge. Each knowledge element was formulated as a logical proposition and was stored in an indexed memory node. Thereby, knowledge elements observed to be jointly accessible in a certain context were put in the same node. That is, the question which knowledge element is accessible in a given context depends on the indexing of nodes. As for the language records, those words playing the role of key words for the activation of a knowledge element were put in the node holding that element. Words that were understood across situations were put in higher nodes.

The knowledge network (i.e., the "cognitive structure") of TERRI as of the first version of the model is depicted in Figure 2. Each of the nodes shown contains a number of (up to 16) expressions constituting language records or knowledge elements.

At the beginning of a dialogue, the system is in "neutral" focus, represented by node *. At that time, the total knowledge base is accessible. Some records in node * serve to understand general principles of the sentence structure, for example, how to evaluate composite questions containing the word "or".

The system contains knowledge about the size comparison of fractions (node F) and of whole numbers (node W). Node W has several knowledge elements forming a basis to understand and to answer such questions in the realm of whole numbers. One element serves to make sure that the knowledge

of this node can only be applied in the intended domain, that is, when whole number questions are posed.

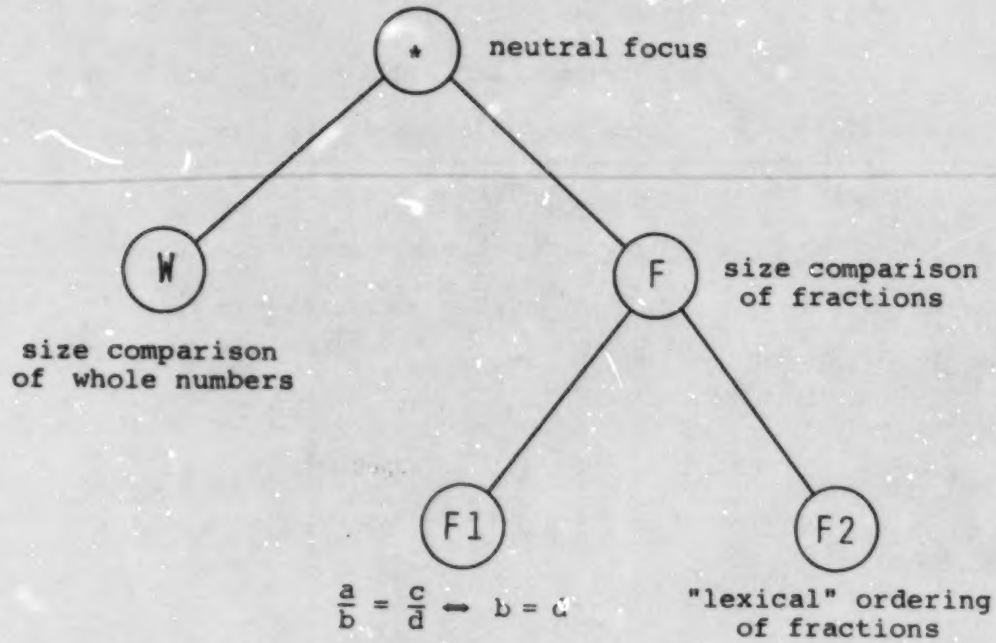

Figure 2 The knowledge network of TERRI

Node F gives access to TERRI's fraction knowledge. Like node W, it contains an element restricting utilization of the knowledge to the intended domain (of fractions). Other elements serve to understand and generate language. TERRI's knowledge of fractions has two subnodes which contain rules corresponding to misconceptions observed with the subject Terri. In some situations, Terri would regard same denominator fractions as equal; in other situations, she would order them in a lexical fashion according to whole number relationships of numerators and denominators (see Part A). In Figure 2, the "key" rules giving rise to such behavior are mentioned. Which

of the two nodes, (F 1) or (F 2), is focussed on, depends on the use of key words in the dialogue. To this end, certain language records are only available in one of either nodes, that is, they are situation-specific.

We use the term visible knowledge to refer to all elements in the node currently focussed on and in all its predecessors. Note that the visible knowledge at a given time is comprised by all nodes on the path from the top node to the focussed node. The domain relevant for processing an input is the visible knowledge. If more knowledge is needed while the processing of an input is in progress, the focus will shift further "down" (toward more specific knowledge), following the links of the network. We use the term accessible knowledge to refer to all successor nodes of the focus node, and we say active knowledge to refer to both visible and accessible knowledge.

Overall, this process of focus adjustment gives rise to a dynamical partitioning of the knowledge base into active knowledge and inactive knowledge (i.e. the complement node set of the active knowledge). For example, if (F 1) is the focus node, then the active knowledge (which is totally visible in this case) consists of the elements in the nodes *, (F), and (F 1), while the knowledge in (W) and (F 2) is inactive in this situation. In this sense, access structures are determined by the topology of the graph of the knowledge network.

If an input cannot be processed on the basis of the knowledge currently active, the focus will shift "up" following the path of visible knowledge and then "down" another branch of the knowledge network if still given one. This process is recursively repeated until, eventually, the system succeeds, or fails, in generating a response.

When a whole working cycle is completed, the focus will remain at the current node and will be the starting focus for the next input. This feature is important since it brings about that the next input, from the beginning, will be interpreted in a certain context ("mind set").

In the the current implementation of TERRI, the semantic representations of the last four questions and answers are kept in short-term memory. By this it is possible to generate cognitive conflicts in the following way. From situational cues in a dialogue the focus may shift to activate a different part of the knowledge base such that a consecutive question is interpreted in a different context. Then, based on the system's memory of the precedent answers, it is possible that an opinion previously taken by the system can be repeated even when it cannot be justified (i.e., derived from active knowledge) in the new context. For example, this is the case when a question is posed a second time but a different wording is used and the relevant knowledge of the two contexts in which the question can be understood is inconsistent. Since the system remembers its previous answer, it will respond as before. A consecutive WHY question, however, will cause the system to acknowledge a conflict since a contradiction is recognized.

3.5 A dialogue with TERRI

In Figure 3, an original protocol of a dialogue conducted with the computer model while it acts on the basis of the above knowledge network is reproduced. Inconsistent behavior is observed in that TERRI's opinion of the size relation between 6 / 15 and 12 / 15 changes several times. At about the middle of the protocol the contents of short-term memory are listed. From these can be seen that TERRI's current opinion of 6 / 15 being less than 12 / 15 is still present when context (F 1) is, cued by the key

words "equal or one less", accessed another time. But in this context, TERRI's answer statement cannot be justified and the recognition of this inconsistency results in a conflict (see Figure 3).

The original dialogue with Terri which was commented on in Part A stems from a situation more complex and probably requires further analysis in terms of attitudinal patterns in the interaction of interviewer and subject. It must not be overlooked that such patterns also contribute to the behavior of a subject. That is, the decisions to act made by Terri certainly cannot be explained on a sole cognitive basis. However, Terri's possible ways to act are constrained by her knowledge repertoire - "false" actions exhibited by Terri must also rely on an application of knowledge she possesses, possibly in an inadequate instance. In this sense, the reproduction of some aspects of Terri's behavior presented here shall serve to discuss the possible contribution that is offered by a logical analysis of Terri's cognitive structures in terms of the model.

It is mentioned that in the current version of the model, there is no technical restriction (except for storage space) to program knowledge networks of a much more complex structure than that of the TERRI model. The complicated question now is how to actually set up a model, by compiling knowledge elements in nodes and by specifying node indices. At present, because of economical reasons, it was decided to reproduce observed behavior with minimal assumptions (i.e., by a network as simple as possible). In the case of Terri this procedure was supported by the fact that relatively distinct knowledge units could be identified from her behavior.

```
I'M TERRI> 6 / 15 AND 12 / 15 , EQUIVALENT?

   DON ' T UNDERSTAND!

I'M TERRI> 6 / 15 AND 12 / 15 , EQUAL?

   YES , 6 / 15 AND 12 / 15 , EQUAL! _______ FOCUS (F 1:*1)

I'M TERRI> WHY?

   BECAUSE (EQ 15 15)! _______ FOCUS (F 1:*1)

I'M TERRI> 12 / 15 AND 6 / 15 , ARRANGE-IN-ORDER!

   6 / 15 AND 12 / 15 ,! _______ FOCUS (F 2:*1)

I'M TERRI> 6 / 15 AND 12 / 15 , WHICH-ONE-LESS?

   6 / 15 AND 12 / 15 , FIRST-LESS! _______ FOCUS (F 2:*1)

I'M TERRI> WHY?

   BECAUSE (LESS 6 12)! _______ FOCUS (F 2:*1)

I'M TERRI> STM!

(STM (! (BECAUSE TRUE (< (6 15) (12 15)) (= (LESS 6 12))) (? (WHY))) .
(STM (! (< (6 15) (12 15))) (? ((OR < >) (6 15) (12 15)))) .
(STM (! (DUMMY (6 15) (12 15))) (! (ORDER (12 15) (6 15)))) .
(STM (! (BECAUSE TRUE (= (6 15) (12 15)) (= (EQ 15 15))) (? (WHY))) .

I'M TERRI> 6 / 15 AND 12 / 15 , EQUAL OR ONE-LESS?

   6 / 15 AND 12 / 15 , ONE-LESS! _______ FOCUS (F 1:*1)

I'M TERRI> 6 / 15 AND 12 / 15 , WHICH-ONE-LESS?

   6 / 15 AND 12 / 15 , FIRST-LESS! _______ FOCUS (F 1:*1)

I'M TERRI> WHY?

   <<< CONFLICT >>> (< (6 15) (12 15)) BUT (= (6 15) (12 15))
   BECAUSE (EQ 15 15)! _______ FOCUS (F 1:*1)

I'M TERRI> 6 / 15 AND 12 / 15 , ONE-LESS?

   NO , 6 / 15 AND 12 / 15 , EQUAL! _______ FOCUS (F 1:*1)

I'M TERRI> WHY?

   BECAUSE (EQ 15 15)! _______ FOCUS (F 1:*1)
```

Figure 3 A dialogue with TERRI

4. A tentative conclusion

Based on the hypothesis of what Terri's current cognitive structure with respect to size comparisons of fractions is like (as expressed in terms of the model), it is possible to obtain a diagnosis of the origins of her malperformance and to speculate about instructional procedures which can bring about progress in Terri's cognitive development. It should be understood that the following remarks are of tentative nature and need further discussion or critique.

As is derived from the model, the instability in Terri's performance is due to the fact that she activates different knowledge units when trying to respond to a question on the basis of her changing interpretation of it. As long as the knowledge network remains as is, Terri's performance cannot become stable since there is no basis in the network providing for that. Thus, the knowledge network needs to be changed. Since there is increasing indication in the literature that knowledge is subject to accretion, it has to be assumed that "false" rules cannot be erased.

The model suggests that performance can be changed if a new node is created which contains information on which type of rule not to use (i.e., intercept application of such a rule). Actually in Terri's case, her notion of the word "order" and the word "equal" needs to obtain a new interpretation in the context of fractions (by "notion" is meant what ways to act are available to her in the context of these words).

We suggest that the effect of alternative frameworks can only be controlled when a new "higher" node is established in the knowledge network which has access to all alternative ways to interpret the data in a situation ("awareness of the range"), together with a rule achieving appropriate

discrimination (inhibition of "false" interpretations).

But one should be cautious with such an argument: The installation of an inhibition in the human mind is not just like "putting in another rule." The "backsliding" phenomenon suggests that access structures may be instable themselves. The "range specifity" phenomenon suggests that stable performance is likely to be restricted to specific situations and does not necessarily generalize to other situations not met before.

On the basis of the model it seems that a possible way to achieve correct performance in a broader range is the following: Many nodes representing specific situations to which certain knowledge is relevant need to become subordinately linked to a node containing rules which can support correct performance. Then those rules would become "visible" for any subordinate node activated. That means, the more specific situations are represented in nodes linked to a node describing an abstract way to act in a class of situations, the greater is the chance that this knowledge becomes activated by a specific situation.

It must be a goal of the insights gained from a cognitive model to reach better ways of teaching such that an optimal use of knowledge is stabilized in broad applicational domains. Computer models could make a significant contribution, through precise analyses of processes of the acquisition and utilization of knowledge. It will need time, and discussion, to further elaborate on these ideas.

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